Exercise 20

Show that $l_1(t) = (1, 2, 3) + t(1, 0, -2)$ and $l_2(t) = (2, 2, 1) + t(-2, 0, 4)$ parametrize the same line.

Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where $\mathbf{m}$ is the direction vector, b is the position vector for a point on the line, and t is a parameter. l_1 is parallel with l_2 because its direction vector is a constant multiple of that for l_2: $-2(1, 0, -2) = (-2, 0, 4)$. Since $l_1(-1) = l_2(1) = (0, 2, 5)$, the two lines travel through the same point. $l_1(t)$ and $l_2(t)$ are therefore just two different ways of parameterizing the same line.